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SINGULARITIES OF THE ROTATING CYLINDRICAL SHELL CONVERGENCE PROCESS

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It is noted in [1, 2] that during the inertial convergence of viscous cylindrical shells the inner shell boundary is arrested upon reaching a certain distance from the axis of symmetry r_{min} whose magnitude depends on the coefficient of viscosity and on the geometric and kinematic shell parameters. This dependence can be utilized to determine the coefficient of viscosity. However, measurements are made difficult because of the small r_{min} for a sufficiently high convergence rate.

Inertial convergence of a rotating cylindrical shell is investigated in this paper with compressibility and viscosity taken into account. Energy transformation and redistribution occurs during convergence of such a shell. The kinetic energy of radial motion is converted into rotation energy and into internal energy of the substance. The energy of the rotational motion goes over into thermal energy due to the viscous friction of the rotating shell layers. A time sets in here when the velocity of radial motion of the inner shell boundary becomes zero, the inner boundary is arrested at a certain distance from the axis of symmetry, after which separation of the shell starts.

The quantity r_{min} depends on the shell geometric dimensions, on the relationship of the kinetic energy of the radial and rotational motion at the initial time, and what is of special interest, on the coefficient of viscosity of the shell material. Depending on the initial data, r_{min} for a viscous rotating shell can turn out to be substantially greater than for a shell of the same dimensions that does not rotate.

The system of equations describing rotating shell motion with viscosity and compressibility taken into account has the following form [3]

$$\begin{gathered} \rho\, du/dt = -\partial P/\partial r + \partial S_{rr}/\partial r + (S_{rr} - S_{\varphi\varphi})/r + \rho\omega^2 r, \\ d\omega/dt = -2u\omega/r + (\partial S_{r\varphi}/\partial r + 2S_{r\varphi}/r)/r, \\ d\rho/dt = -\rho(\partial u/\partial r + u/r),\ dr/dt = u, \\ de/dt = -P\, d1/\rho/dt + (S_{rr}^2 + S_{\varphi\varphi}^2 + 2S_{r\varphi}^2)/2\rho\mu, \\ P = P(\rho, e),\ S_{rr} = \mu(2\partial u/\partial r - u/r)2/3, \\ S_{\varphi\varphi} = (2/3)(\mu(2u/r - \partial u/\partial r)),\ S_{r\varphi} = \mu r\,\partial\omega/\partial r, \end{gathered} \tag{1}$$

where ω is the angular rotation, μ is the coefficient of viscosity, S_{ij} are the viscous stress tensor deviator components in an x, r, φ coordinate system (ox is the axis of rotation), and the remaining notation is standard.

Heat conductivity and second viscosity effects are not taken into account in this formulation. For the case of condensed media under consideration such an approximation is justified. The heat conductivity effects are excluded since the thermal relaxation time $\tau = \ell^2/\chi$ is substantially greater than the characteristic times of the process [3, 4] (ℓ is the characteristic dimension and χ is the thermal diffusivity coefficient). The contribution of second viscosity to the global part of the stress tensor is small compared with the pressure [5].

The system (1) was solved numerically by a finite-difference method using the method of splitting according to physical processes [6, 7]. The solution of the system (1) was

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separated into two stages at each step of the integration over time. The first stage in the calculations is the solution of the subsystem describing the motion of a non-rotating shell (the equations of this stage are obtained from the system (1) by discarding all terms containing the angular velocity). Computation of the terms discarded in the first stage is performed in the second stage. Explicit difference schemes obtained by an integro-interpolation method are utilized. The difference schemes are realized in a programmed complex [7].

The counting method and the program were tested in a number of problems that have analytic solutions. In particular, convergence of a rotating cylindrical shell from an ideal fluid and convergence of a viscous shell without rotation were computed [2]. Good agreement is obtained between the computed and analytical results. For example, the discrepancy between the computed and analytical results is less than 1% in the radius of shell detent.

Numerical investigation of the inertial convergence process was performed in an example of a cylindrical shell with inner radius R_0 = 10 cm and thickness ΔR_0 = 0.3537 cm. The equation of state was used in the Mie–Grüneisen form [5]

$$P = \rho_0 c_0^2(\delta^n - 1)/n + \Gamma\rho(e - e_x),$$
$$e_x = c_0^2[(\delta^n - n\delta)/(n-1) + 1]/\delta n,\ \delta = \rho/\rho_0$$

with the parameters ρ_0 = 7.85 g/cm^3; c_0 = 4.6 km/sec; n = 3; Γ = 0.67. At the initial time u_0 = 2 km/sec and an angular velocity $\omega_0 = 2\cdot10^3$ sec^{-1}, which corresponds to the linear velocity $u_{rot} = \omega_0 R_0$ = 0.2 km/sec were given in the shell.

Given as boundary conditions on the outer and inner shell surfaces are $\sigma_{rr} = \sigma_{r\varphi} = 0$. Values of r_{min} determined in computations with a different number of counting points (20, 40, 60) in the domain and different μ are presented in Table 1. Values of r_{min} obtained by linear extrapolation to an infinite number of counting points are given in the last column of Table 1 and in Fig. 1. The dependence of the shell turning radius on the time is shown in Fig. 2 (the number of the curve corresponds to the computation number from Table 1).

A maximal u_{rot} = 6.6 km/sec is achieved in the computation with zero viscosity at the time of detent ($r_{min} \approx$ 0.3 cm). The velocity calculated on the basis of the moment of momentum conservation law is $u_{rot} = 0.2(10/0.3) \approx 6.67$ km/sec. At the time of turning the shell material is in the compressed state; the maximal compression is $\delta_{max} = \rho/\rho_0 = 1.5$. Consequently, r_{min} = 0.3 cm from the computations is noticeably greater than the r_{min} for an incompressible shell with the same initial parameters (r_{min} = 0.08 cm) calculated by the formula we determined

$$r_{\min} = \sqrt{2R\Delta R}\Big/\sqrt{\exp\left(2u_0^2\Delta R_0/\omega_0^2 R_0^3\right) - 1}.$$

As is seen, r_{min} diminishes as the coefficient of viscosity increases (see Figs. 1 and 2). This is explained by the fact that viscous friction of the shell layers results in diminution of the rate of inner layer rotation and of the centrifugal force acting on the shell layer and assuring its rotation. The thermal pressure grows because of rotation energy

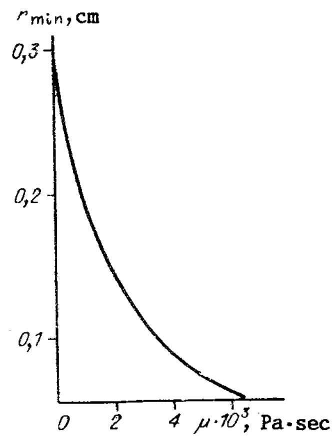

Fig. 1

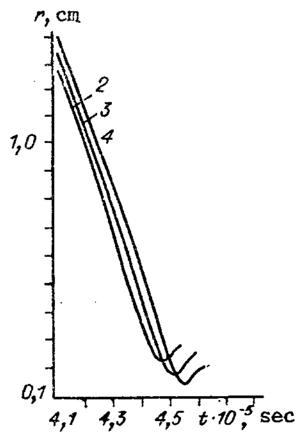

Fig. 2

TABLE 1

Computation No.	$\mu \cdot 10^3$ Pa·sec	Number of points			
		20	40	60	∞
1	0	0,348	0,326	0,318	0,304
2	2	0,22	0,182	0,17	0,144
3	4	0,171	0,13	0,119	0,089
4	6	0.14	0.1	0.086	0.06

dissipation, resulting in expansion of the inner layers and additional diminution of the shell turning radius. The maximal magnitude of the pressure reaches ~140 GPa for $\mu = 6 \cdot 10^3$ Pa·sec. These parameters are realized near the time of inner shell boundary turning. The maximal rate of inner surface rotation for such a shell is ~3.5 km/sec. Let us note that in conformity with the momentum conservation law, the rate of rotation would be ~30 km/sec in the absence of viscosity. In the absence of rotation, for a given shell determined by the formula from [2], the r_{min} would equal $\approx 10^{-14}$ cm, i.e., is not accessible to recording. The dependence of the turning radius of a rotating shell on μ, detected in the computations, is illustrated in the Table 1 and in Figs. 1 and 2.

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